BMath Algebra-I
End-Semestral Exam
2016-2017

Time: 3 hrs
Max score: 100

Answer all questions.

(1) (a) Describe the automorphism group of Klein 4-group.
(b) Let G be a group of order 203. Prove that if H is a normal subgroup of order 7 in G, then G is contained in $Z(G)$. Deduce that G is abelian in this case. (10+10)

(2) (a) Write down explicitly all the Sylow subgroups of A_5.
(b) Show that a group of order 300 is not simple. (10+10)

(3) (a) Describe commutator subgroup of a group G. Show that the commutator subgroup of S_n is A_n, $\forall\ n \geq 3$.
(b) Prove that a finite abelian group is the direct product of its Sylow subgroups. (8+12)

(4) (a) Prove that a group of order 12 either contains a normal subgroup of order 3 or is isomorphic to A_4.
(b) Show that A_n is the only non-trivial proper normal subgroup of S_n for $n \geq 5$. (You may use simplicity of A_n for $n \geq 5$). (10+10)

(5) (a) Define semi-direct product of two groups.
(b) Let p, q be primes, $p < q$ and G a group of order pq. Prove that
i) If p does not divide $q - 1$, then G is cyclic.
ii) If p divides $q - 1$, then there exists a non-abelian group of order pq. (6+14)

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